Eastwood Village Primary School

Calculation Policy

Reviewed: September 2018

Next Review: June 2020

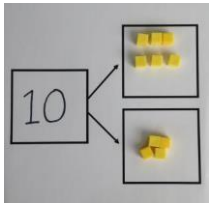
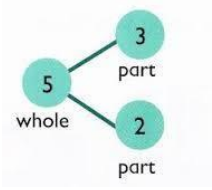
CALCULATION POLICY

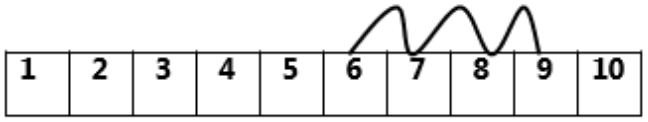
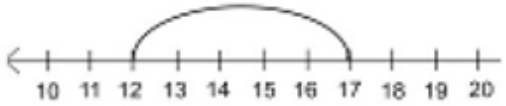
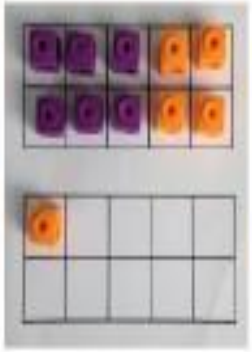
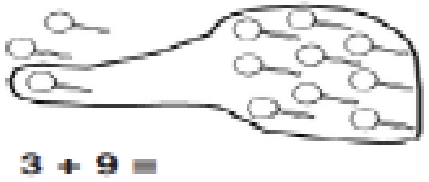
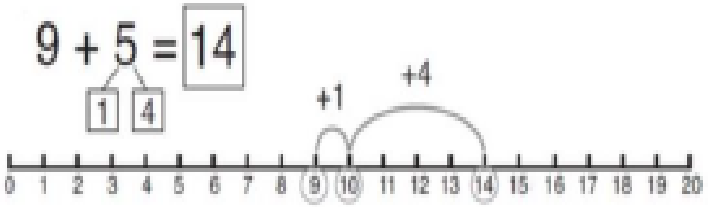
This policy lays out the expectations for both mental and written calculations for the four number operations and has been created to support the teaching of a mastery approach to mathematics. This is underpinned by the use of models and images that support conceptual understanding and this policy promotes a range of representations to be used across the primary years. Mathematical understanding is developed through use of representations that are first of all concrete (e.g. counters and multilink cubes), and then pictorial (e.g part whole) to then facilitate abstract working (e.g. formal written methods). This policy is a guide through an appropriate progression of representations and if at any point a pupil is struggling with the abstract, they should revert to familiar pictorial and/or concrete materials/representations as appropriate. As children move through the different stages, representations should be modelled alongside each other to ensure a secure understanding is maintained. Children should only move onto the abstract method when they have a secure understanding of the two former methods. This policy should be used alongside the more detailed written methods document.

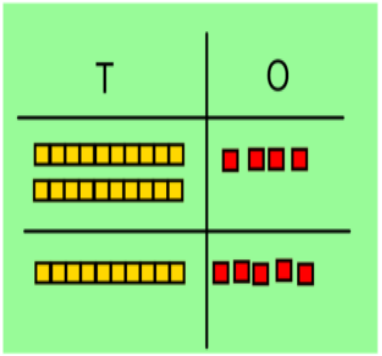
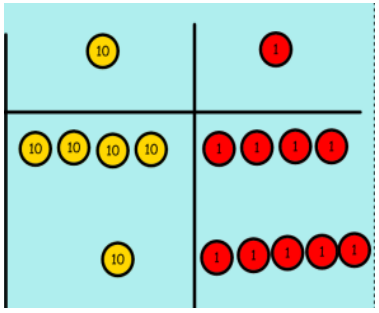
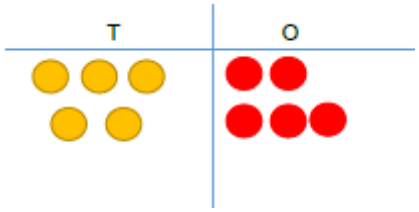
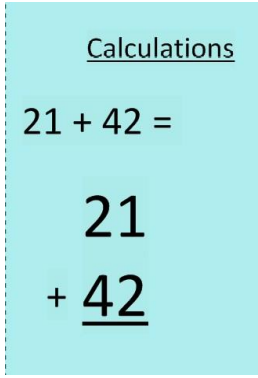
Although this policy sets out the main methods of mental and written calculations to be taught, it has been appended with a list of recommendations and effective practice teaching ideas aimed at informing and enhancing teaching across all the primary phases. Many of these ideas come from the NCETM's Calculation Guidance document (published October 2015) which is intended to sit alongside a school's calculation policy.

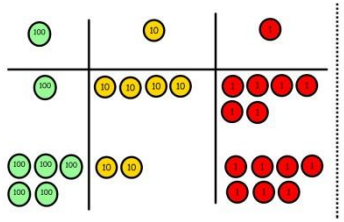
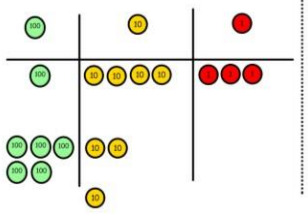
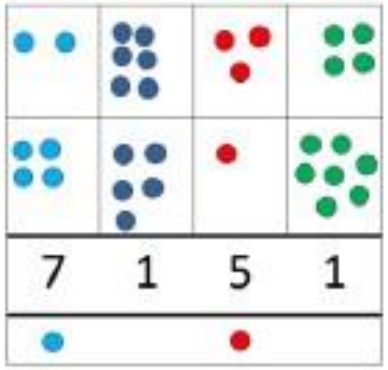
Progression in Calculations

Addition

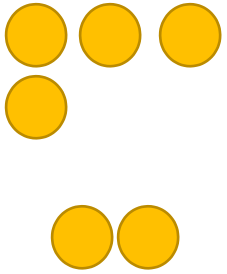
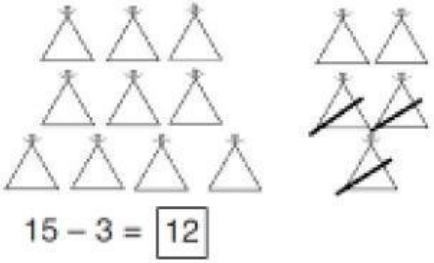
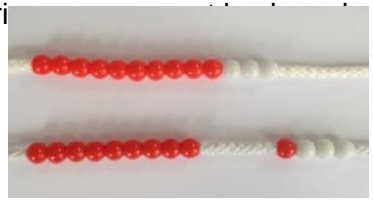
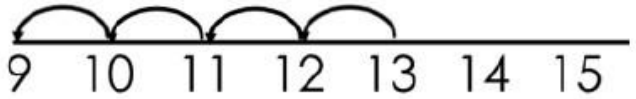
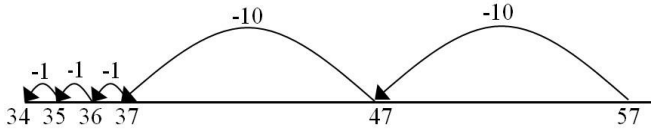
Objective and Strategies	Concrete	Pictorial	Abstract
Stage 1 Counting a set of objects. This can include counting using fingers.			 Children relate the number of objects to the numeral
Stage 2 Combining two parts to make a whole: part-whole model	 Use cubes to add two numbers together as a group or in a bar.	 Use pictures to add two numbers together as a group.	4 + 3 = 7 10= 6 + 4 Use the part-part whole diagram as shown above to move into the abstract.

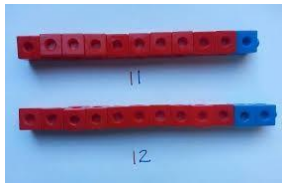
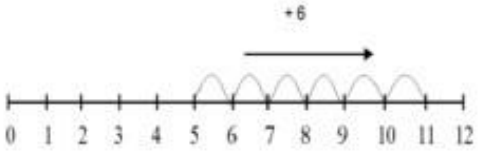
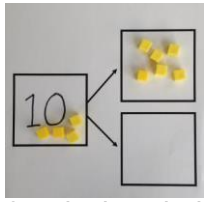
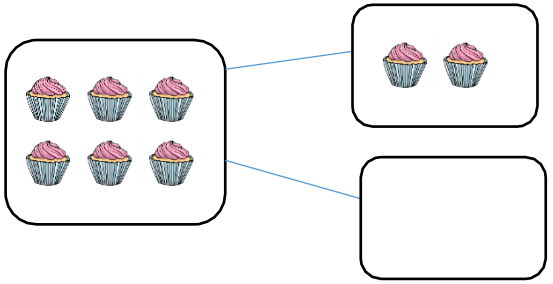
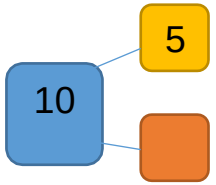
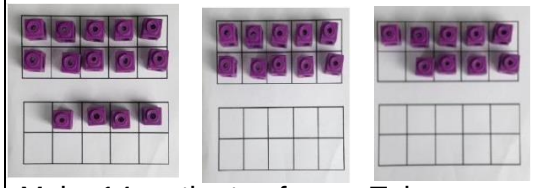
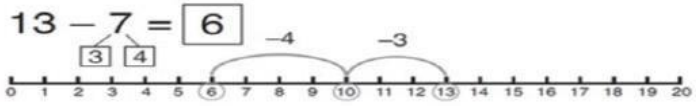
Stage 3 Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	Counting on in jumps of 1 using a number line with numbers on it. For 6 + 3 = 9:  This can also be done in bigger jumps or 1 big jump to find the answer. For 12 + 5 = 17:  	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Stage 4 **Regrouping to make 10 so the calculation is easier**	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10.  	7 + 5 = 7 + 3 + 2 = 12 If I have seven, how many of 5 do I need to add to make 10. How many more do I still need to add on?

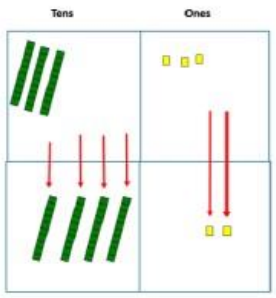
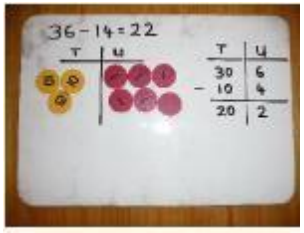
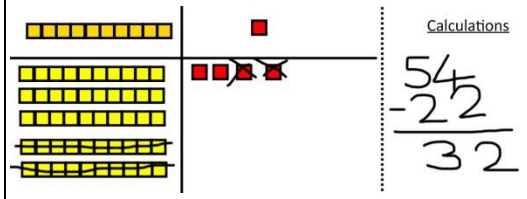
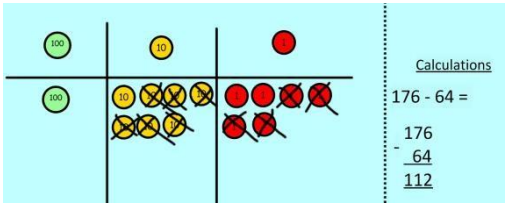
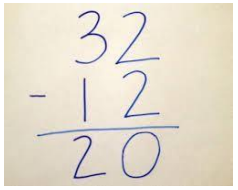
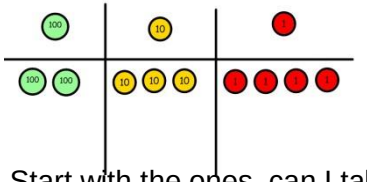
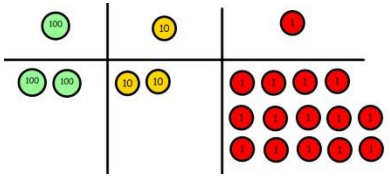
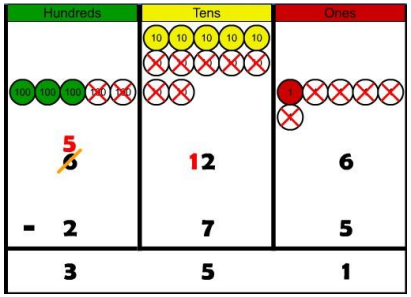
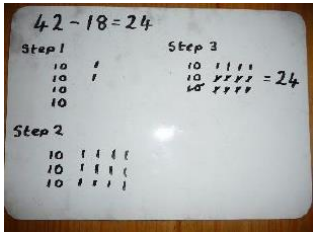
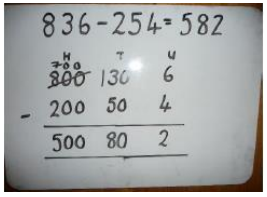
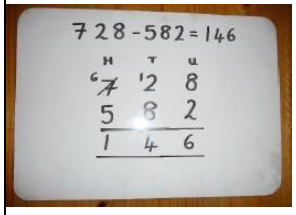
Stage 5 Column addition without regrouping	24 + 15=39 Partition the numbers into tens and ones using the Base 10 before moving onto place value counters. Add together the ones first then add the tens. Finally add the two totals together.   	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 32 + 23 = 55  	 Record the calculations vertically adding the column of ones then the column of tens.

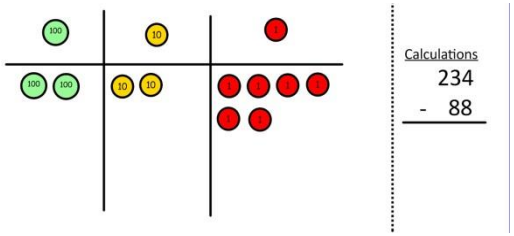
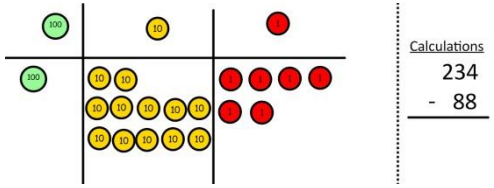
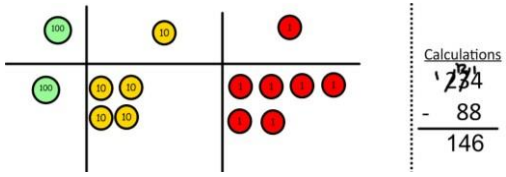
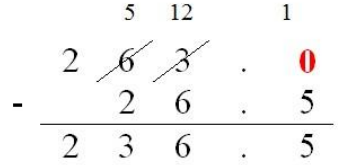
Stage 6 Column addition with regrouping	Make both numbers with place value counters.  146 + 527 In this case, adding the ones gives us 13 which is made up of 10 and 13.  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 536 + 85 621 1 1 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2

Subtraction

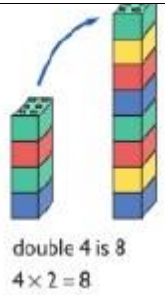
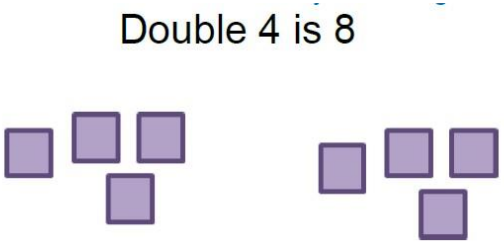
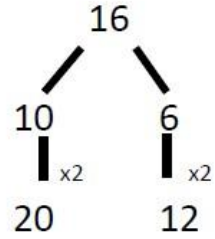
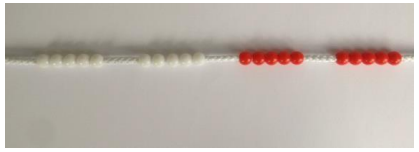
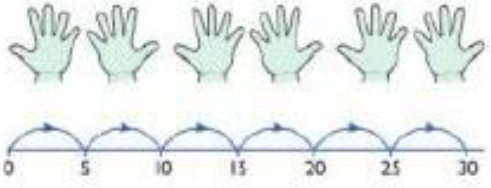
Objective and Strategies	Concrete	Pictorial	Abstract
Stage 1 Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6 Although number sentences are recorded in the concrete and pictorial methods, children are introduced to them on their own while encouraging them to mentally take away ones.
Stage 2 Counting back	Make the larger number in your subtraction. Move the beads along your bead stri in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

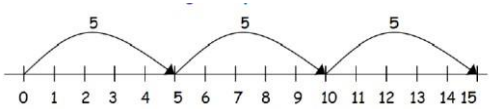
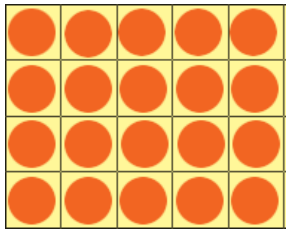
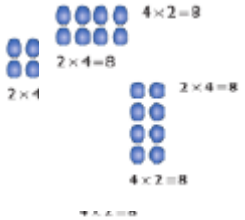
Stage 3 Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference	 Count on to find the difference.	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Stage 4 Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Stage 5 Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

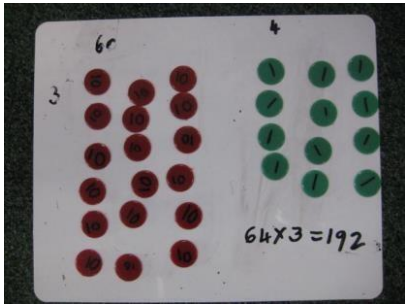
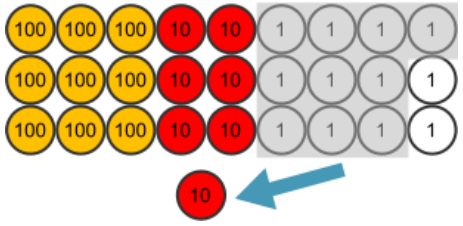
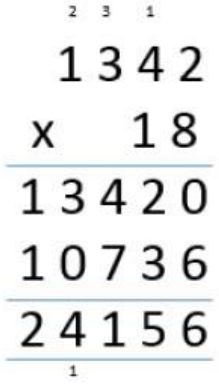
Stage 6 Column method without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	 Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	47 - 24 = 23 40 + 7 - 20 + 4 20 + 3 This will lead to a clear written column subtraction. 
Stage 7 Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones. 	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method.

	Now I can subtract my ones.  Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		This will lead to an understanding of subtracting any number including decimals. 

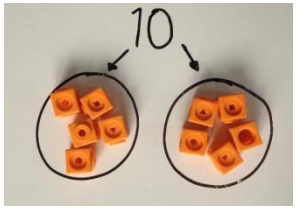
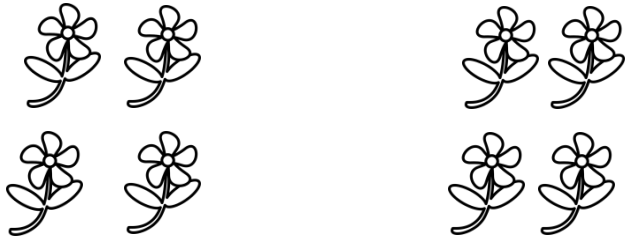
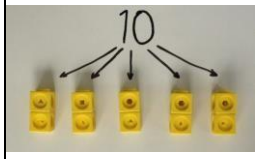
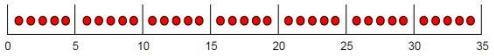
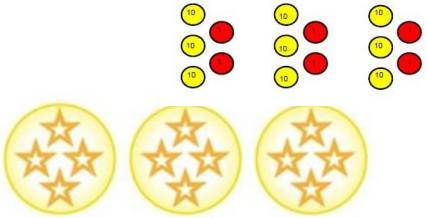
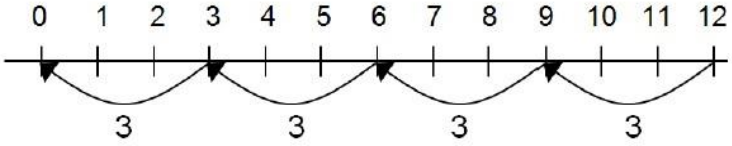
Multiplication

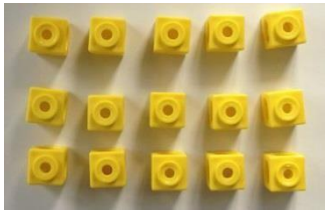
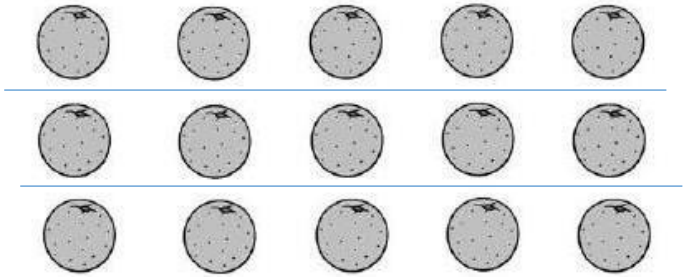
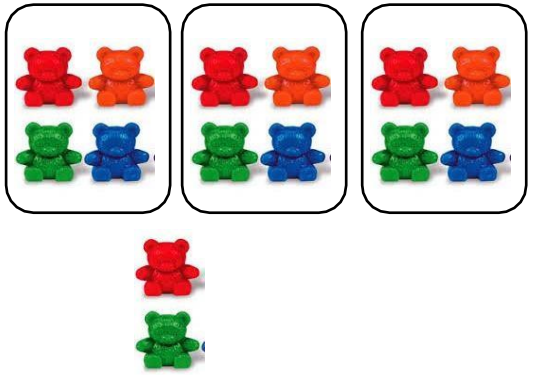
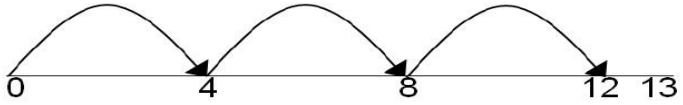
Objective and Strategies	Concrete	Pictorial	Abstract
Stage 1 Doubling	 vities to show how to double a number.	Draw pictures to show how to double a number. 	 Partition a number and then double each part before recombining it back together.
Stage 2 Counting in multiples	 Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

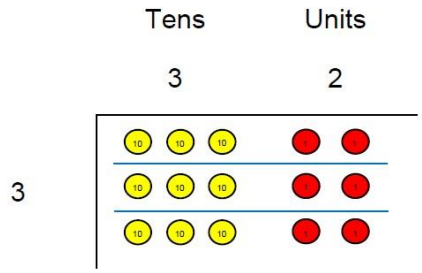
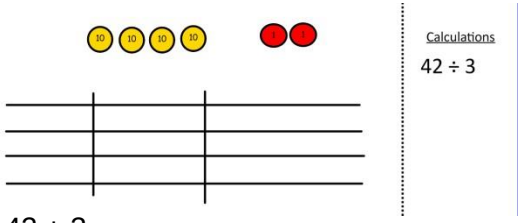
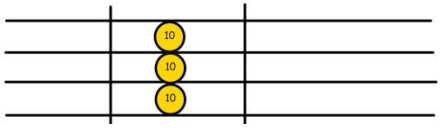
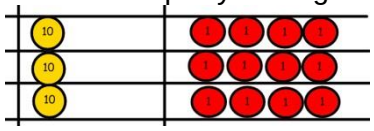
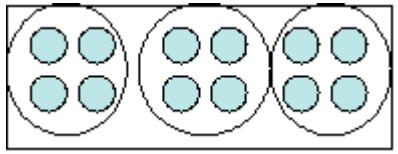
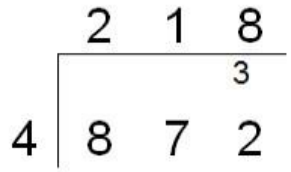
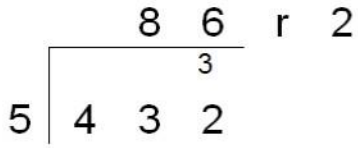
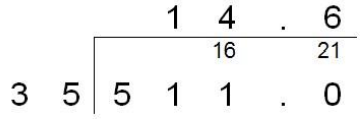
Stage 3 Repeated addition	 Use different objects to add equal groups. Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there? 2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. This then leads to writing the related multiplication sentences e.g 2 x 5 = 10
Stage 4 Arrays – showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences. 4 x 6 = 24 Begin to look at arrays in different orientations to make the link between, for example, 5 x 3 = 15 and 3 x 5 = 15	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

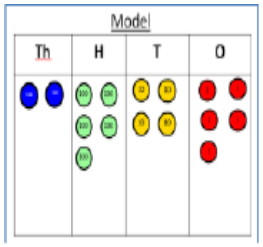
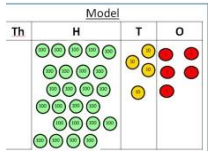
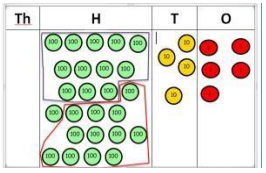
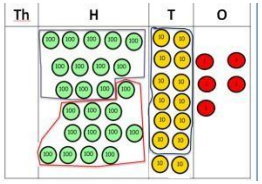
Stage 5 Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  4 x 3 = 12 60 x 3 = 180 64 x 3 = 192 It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below. The idea of exchanging will support them in moving on to a more compact method: 3 x 324  	As with stage 4, children can represent the work they have done with place value counters in a way that they that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking.	If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 Remind children about the importance of lining up numbers clearly in columns. 

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Stage 1 Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Stage 2 Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Stage 3 Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of the multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Stage 6 Long Division	14 ÷ 3 = Divide objects between groups and see how much is left over 	Draw dots and group them to divide an amount and clearly show a remainder. Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 13 ÷ 4 = 3 r1  As knowledge of place value improves, children can begin to jump in multiples of 10: 92 ÷ 3 = 30 r2	Complete written divisions and show the remainder using r. Children use knowledge of times table facts to quickly calculate divisions involving remainders. Go on to combining knowledge of times tables with place value to calculate more difficult divisions. For example: 137 ÷ 4 = 34 r 1

Stage 5 Short division	 Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately. 

Stage 6 Long division	 2544 ÷ 12 How many groups of 12 thousands do we have? None Exchange 2 thousand for 20 hundreds.  How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.  Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2  Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2	Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books. Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.	Children should already understand addition and multiplication with a column method before attempting with division. 0 3 1 8 *r* 5 20 ⟌ 6 3 6 5

Appendix

Listed below are a range of recommendations and teaching ideas aimed at informing and enhancing the teaching of primary mathematics:

1. Developing children's understanding of using the = sign

The = symbol is an assertion of equivalence. If we write 3 + 4 = 6 + 1 then we are saying that what is on the left of the = symbol is equivalent to what is on the right of the symbol. But many children interpret = as always being an instruction to work out the value of a calculation. This is as a result of always seeing it used as follows:

3 + 4 =

5 × 7 =

16 – 9 =

If children only think of = as meaning "Work out the answer to this calculation" then they are likely to get confused by empty box questions such as:

3 + ☐ = 8 and are very likely to struggle with even simple algebraic equations, such as: 3y = 18. This can be overcome by doing the following:

Vary the position of the = symbol e.g. 24 = 4 x 6

Include lots of empty box problems e.g. 12 - ☐ = 4; ☐ x 6 = 24

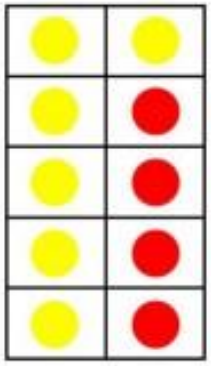
Teach inequality alongside equality e.g. 5 + 9 ☐ 3 x 5 (< > or =?)

2. Recognising the actual value of ones, tens, hundreds etc. in a number

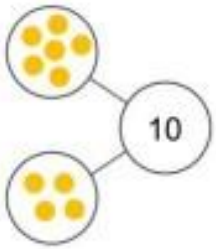
Many children are able to recognise the value of each digit in a number like 347 but find it harder to explain, for example, how many tens there are in 347. Once they are able to recognise that there are 34 tens (rather than 4 tens), it makes it much easier to be able to carry out a calculation such as 347 + 30 as they are adding 3 tens to the 34 tens. Traditionally, children often struggle when tackling a calculation involving crossing over a hundred e.g. 293 + 10 but using this method takes much of the difficulty away as they only need to add 1 ten to the 29 tens to give 30 tens and an answer of 303. It is equally effective when subtracting e.g. for 112 – 20, we subtract 2 tens from the 11 tens to leave is with 9 tens and an answer of 92.

3. Reasoning about mathematical relationships

Children need to be exposed to images and structures that help them to make links between inverse operations from an early age



6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4



6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4

Part Whole Model

Opportunities should be taken wherever possible to demonstrate how children can use what they already know to work out a related fact e.g.:

.- if 6 + 4 = 10, then 6 tens + 4 tens = 10 tens i.e. 60 + 40 = 100

- If you know 3 + 5, you can use this to work out 23 + 5

4. Developing children's fluency with number skills

Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. Research has shown that spending a short time every day on these basic facts quickly leads to improved fluency. At East Dene Primary School, this will be done for 10 minutes at beginning of every Math's lesson. This will be done through Math's Passports.

5. Developing fluency in mental calculations (The Magic 10)

Although the Magic 10 already has a place in this calculation policy, it is worth emphasising the importance of this approach. Children who learn to 'make 10' to create an easier calculation are able to develop mental fluency and an ability to look for patterns. Using knowledge of number bonds that make 10, they can see that 9 + 6 = 9 + 1 + 5 = 10 + 5 = 15